DESIGN AND GAIT ANALYSIS OF A TWO-LEGGED MINIATURE ROBOT WITH PIEZOELECTRIC-DRIVEN FOUR-BAR LINKAGE

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ABSTRACT
This paper presents the design and development of a new type of piezoelectric-driven robot, which consists of a piezoelectric unimorph actuator integrated as part of the structure of a four-bar linkage to generate locomotion. The unimorph actuator replaces the input link of the four-bar linkage and motion is generated at the coupler link due to the actuator deflection. A dimensional synthesis approach is proposed for the design of four-bar linkage that amplifies the small displacement of the piezoelectric actuator at the coupler link. The robot consists of two such piezo-driven four-bar linkages and its gait cycle is described. The robot speed is derived through kinematic modelling and experimentally verified using a fabricated prototype. This result will be important for developing a motion planning control strategy for the robot locomotion, which will be part of future work.

1 INTRODUCTION
Miniature mobile robots offer many advantages due to their small size. They are able to enter and move in tight spaces that are unreachable by big robots. In addition, due to their light weight, they can be easily transported and deployed in hard-to-reach environments. This has vast applications in surveillance, exploration, or search and rescue operations. Piezoelectric actuators, due to their small size, are a potential enabling technology for driving miniature robots. Such actuators have been studied by a variety of researchers for use in robot locomotion and a variety of designs have been proposed.

For instance, Sahai et al. [1] uses two clamped-free piezoelectric bimorph actuators in conjunction with a hexapod structure to move a robot with alternating tripod gait. Ho and Lee [2] use two pieces of Lightweight Piezoceramic Composite Curved Actuator (LIPCA) [3] to drive four legs of a robot with bounding gait locomotion. Baisch [4] uses six piezoelectric actuators developed by Wood et al. [5] for a quadrupedal structure where each contralateral pair of legs were driven by three piezoelectric actuators arranged to generate lift and swing through a linkage
mechanism. Avirovik et al. [6] uses eight piezoelectric bimorphs to drive a millipede-inspired miniature robot. Hariri et al. [7] uses two thin piezoelectric ceramic patches to generate a traveling wave on a robot with beam structure. Rios et al. [8] utilizes piezoelectric bimorph benders to drive a six-legged robot, where each leg consists of two bimorph benders mounted side-by-side joined at the tip by a flexure and an end-effector to generate walking motion.

The design of piezo-driven walking robots, in general, consists of a structural body with either active or passive legs. The former uses legs made of piezoelectric materials to actively drive the robot, but it easily faces issues of improper gait motion if the legs are not properly assembled [9]. However, they have the advantage of modeling simplicity over passive legs design due to the direct interaction between the piezo legs and the ground. The latter type uses rigid or semi-rigid legs attached to a piezoelectric actuator directly or through a mechanism to create locomotion. Several robots have been designed based on this approach. Some are over-actuated where the number of piezoelectric actuators required to control the motion exceeds the degrees of freedom [10, 11], while others are under-actuated [12, 13].

A significant challenge that must be addressed in developing piezoelectric-driven robots is the limited displacement of the piezoelectric actuators [14]. Piezoelectric cantilevers usually have sub-mm displacements and thus require amplification through linkage mechanism in order to achieve larger stroke [15]. The piezoelectric miniature robots developed by Wood et al. [16] and Goldfarb et al. [17] both use five-bar linkage to amplify the leg motion. Sitti [18] uses multiple four-bar linkages to enable stroke amplification of the wings of a piezo-driven micromechanical flying insect. It is important to note that while most of the above robotic systems show satisfactory performance, the geometric parameters of the linkages are chosen arbitrarily or through some optimization process to obtain an optimal stroke amplification. There still does not exist a technique for a task-oriented design of piezo-driven mechanism that achieves a required stroke amplification or gait trajectory.

In this paper, we describe our design methodology and show how it can be applied into the development of a two-legged piezoelectric mobile robot for bi-directional motion. This is our first step towards the development of novel piezo-driven robotic systems that can potentially re-create biological motion with simplified gait controls. To design the robot, we offer an approach to design four-bar linkages as a constrained robotic system, that achieves a required stroke amplification given the limited displacement of the input link. To predict the robot motion, kinematic analysis is perform to estimate the speed of the robot based on an input voltage and frequency. A prototype is fabricated based on the design, and experimental results for the speed versus applied voltages and frequencies are given and compared with our kinematic model to verify its validity.

2 DIMENSIONAL SYNTHESIS OF PIEZO-DRIVEN FOUR-BAR LINKAGE WITH REQUIRED STROKE AMPLIFICATION

Figure 1 shows the 3D model of our proposed robot design. It consists of two cantilever piezoelectric unimorph actuators, a central body, and two pairs of four-bar linkage legs at opposite ends of the robot’s body. Each of these actuators drives the robot in one particular direction. A cantilever piezoelectric unimorph actuator consists of a layer of piezoelectric material and a layer of elastic material bonded together where one end is fixed and the other end is free. When an electric field is applied to the piezoelectric material in the direction of the poling axis, the piezoelectric material will expand. On the other hand, when an electric field is applied to the piezoelectric material in the opposite direction of the poling axis, the piezoelectric material will contract. The expansion and contraction of the piezoelectric layer results in bending deflection due to the stress difference between the two layers. Hence, if we replace this with the input link of a four-bar linkage, articulated motion can be created at the coupler link, as depicted in Fig. 2, moving the robot.
FIGURE 3. The pseudo-rigid-body model of the unimorph actuator OA, constrained by an RR chain CB, to form a four-bar linkage.

To design the robot’s leg, we consider how a unimorph actuator can be mechanically constrained by an RR chain to form a one degree-of-freedom four-bar linkage as shown in Fig. 3. This requires the designer to select the various link dimensions of the pseudo-rigid-body model of the unimorph actuator for task identification and geometric synthesis. To identify the task for dimensional synthesis, the vibration motion of the actuator is analyzed to establish its joint limits $\theta$.

2.1 Joint Limit Identification

To identify the actuator joint limits, its equation of motion is derived based on Euler-Bernoulli hypothesis that the stress in the x-direction is uniaxial, the deformation is small, and the cross section remains perpendicular to the neutral axis after deformation. It is also assumed that the electric field is uniformly distributed in the z-direction, i.e. $E = E_z(x,t)$. Then, the displacement field becomes

$$
\mathbf{u} = \begin{cases}
  u_x(x,y,z,t) \approx -(z - z_n) \partial_x w(x,t) \\
  u_y(x,y,z,t) = 0 \\
  u_z(x,y,z,t) \approx w(x,t)
\end{cases}
$$

where the geometric parameters for the system are given in Fig. 4 and $w(x,t)$ is the transverse displacement of the actuator about its neutral axis $z_n$, whose location can be obtained using

$$
  z_n = \frac{1}{2} \frac{cm_t^2 + cp_{pf}^2 + 2cp_ppt_m}{cm_t + cp_p},
$$

where $c_p = \frac{1}{s_{xx}^E}$, $c_m$ is the Young’s modulus of the elastic layer, and $s_{xx}^E$ is the elastic compliance of the piezoelectric layer. See Ballas [19].

It was found in Hariri et al. [20] that in static operation, the equation of motion becomes equivalent to the static beam equation

$$
(EI)_{eq} \frac{\partial^2 w(x)}{\partial x^2} = -q e_p E_z.
$$

where

$$(EI)_{eq} = (I_p c_p + I_m c_m),
$$

$$
I_m = b \int_0^{t_m} (z - z_n)^2 dz = b \frac{1}{3} \left( (t_m - z_n)^3 + z_n^3 \right),
$$

$$
I_p = b \int_{t_m}^{t_p + t_m} (z - z_n)^2 dz = b \frac{1}{3} \left( (t_p + t_m - z_n)^3 - (t_m - z_n)^3 \right),
$$

$$
q = b \int_{t_m}^{t_p + t_m} (z - z_n) dz = \frac{b}{2} \left[ (t_p + t_m - z_n)^2 - (t_m - z_n)^2 \right],
$$

$e_p = \frac{dz}{s_{xx}^E}$ and $d_{cx}$ is the piezoelectric charge constant.

Now, with fixed-free boundary conditions, the transverse displacement of the actuator is given by

$$
w(x) = \frac{1}{2} k x^2,
$$

where $k = \frac{-q e_p E_z}{(EI)_{eq}}$. 

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The piezoelectric material will be actuated at its first resonant frequency to exploit maximum transverse deflection. The first resonant frequency of the actuator can be calculated as

\[ f_1 = \frac{(\beta_1 l)^2}{2\pi l^2} \sqrt{\frac{(EI)_{eq}}{(\rho A)_{eq}}}, \]  

where \( \beta_1 l = 1.875 \), \((\rho A)_{eq} = b(\rho_p t_p + \rho_m l_m)\), and \( \rho_p \) and \( \rho_m \) are the density of the piezoelectric and elastic material respectively. See Graff [21].

To determine the joint limits, we leverage on the fact that the piezoelectric deflection is very small. Hence, small angle approximation applies and the angular displacement of the piezoelectric cantilever flexes downwards and \( \theta_{\text{min}} \) corresponds to the input angle when the piezoelectric cantilever flexes upwards. Then, the range of angular displacement of the input crank is given by

\[ \theta_{1, \text{min}} = \theta_{1,0} - \zeta, \]
\[ \theta_{1, \text{max}} = \theta_{1,0} + \zeta, \]

where \( \theta_{1, \text{max}} \) corresponds to the input angle when the piezoelectric cantilever flexes downwards and \( \theta_{1, \text{min}} \) corresponds to the input angle when the piezoelectric cantilever flexes upwards.

2.2 Stroke Amplification through Task Specification

To specify the stroke amplification requirements at the leg, we generate the positions reached by the unimorph actuator using the kinematics equation of a planar 2R chain, taking into account its joint limits as calculated above and the required amplification factor at the leg. The kinematic equations of a planar 2R chain equate the 3 x 3 homogeneous transformation \([T]\) between the end-effector and the base frame to the sequence of local coordinate transformations around the joint axes and along the links of the serial chain

\[ [T] = [G][Z(\theta_i)][X(a)][Z(\theta_2)][H]. \]  

The parameters \( \theta_i \) define the movement of each joint, and \( a \) defines the length of the link. The transformation \([G]\) defines the position of the base \( O \) of the chain relative to the world frame, and \([H]\) locates the task frame \( M \) relative to the end-effector frame. See Fig. 3.

The required stroke amplification of the unimorph actuator to the leg can be specified by introducing a scaling factor \( \alpha \), and we use the following relationship to define its joint limits

\[ \theta_{2, \text{min}} = \theta_{2,0} - \alpha \zeta, \]
\[ \theta_{2, \text{max}} = \theta_{2,0} + \alpha \zeta. \]  

We now choose five joint parameters from this range to obtain five task positions using Eq. (8) for the synthesis of an RR chain to yield a four-bar linkage.

2.3 Synthesis of an RR Constraint

The synthesis of an RR link is well known [22] and to design an RR link that reaches the above set of five task positions, we solve the following set of design equations

\[ (B^2 - C) \cdot (B^2 - C) - (B^4 - C) \cdot (B^4 - C) = 0, \]
\[ (B^3 - C) \cdot (B^3 - C) - (B^1 - C) \cdot (B^1 - C) = 0, \]
\[ (B^4 - C) \cdot (B^4 - C) - (B^2 - C) \cdot (B^2 - C) = 0, \]
\[ (B^5 - C) \cdot (B^5 - C) - (B^3 - C) \cdot (B^3 - C) = 0, \]

where \( C \) denotes the fixed pivot and \( B \) denotes the moving pivot at the \( i \)th position. The moving pivot is related to its first position by

\[ B^i = [T_i][T_i]^{-1} B^1, \quad i = 2, \ldots, 5. \]  

3 ROBOT’S GAIT CYCLE FOR LOCOMOTION

To drive our synthesized piezoelectric robot, we use a gait cycle as denoted in Fig. 6(a)-(g). One unimorph actuator will be operated to drive the robot to move in one particular direction using a sinusoidal input. This cycle was chosen purely for...
simplicity so as to establish a proof-of-concept that controllable independent motion can be achieved. We intend to explore more complex coordinated gait locomotion as part of future work. In this gait cycle explanation, the left unimorph actuator is the active actuator. This moves the robot to the left. We assume that the leg in contact with the ground does not slip and double-leg flight phase does not occur, i.e. at least one of the robot’s legs always touches the ground.

The gait cycle starts with the left unimorph actuator flexing maximally upwards at time $t = 0$ and both robot’s legs on the ground (Fig. 6(a)). From this configuration, the piezoelectric layer begins to expand and starts to deflect the unimorph actuator downwards toward its neutral position at $t = T/4$ (Fig. 6(b)) and to its maxima at $t = T/2$. This pulls the robot’s body upwards and leftwards by pivoting about point $L$ as shown in Fig. 6(c). This is followed by an instantaneous clockwise rotation of $\gamma'$ about $L$ due to gravity, until the robot’s right leg touches the ground again at $L'$ (Fig. 6(d)). To complete the gait cycle, the unimorph actuator starts to deflect upwards again, passing through its neutral position at $t = 3T/4$ (Fig. 6(e)) and its minima position at $t = T$ (Fig. 6(f)); this time supported at $L'$ as shown in Fig. 6(e) to (f). Similarly, immediately after this, the whole robot will experience an anti-clockwise rotation of $\gamma$ about $L'$ due to gravity, until the left leg touches the ground again at $L$. This completes the gait cycle to its original state as shown in Fig. 6(g). The cycle then repeats moving the robot to the left.

4 GAIT ANALYSIS & MATHEMATICAL MODELING
To mathematically model the robot’s motion based on the gait cycle, the non-actuated leg will be treated as a rigid body with the robot’s body. To analytically calculate the robot’s displacement after one full gait cycle, we will analyze the gait cycle in two halves: the front leg stance half cycle (Fig. 6(a) to (d)) and the back leg stance half cycle (Fig. 6(d) to (g)).

4.1 Front Leg Stance Gait Analysis
To determine the robot displacement during its first half of gait, consider a leg kinematic model as shown in Figure 7. Let the fixed and moving pivots of the input link be $O$ and $A$, and the fixed and moving pivots of the output link be $C$ and $B$, respectively. Denote a fixed base frame $F$ with its origin at $O$ and

![Figure 6](image-url)  
**Figure 6.** Gait cycle model of the piezoelectric robot.

![Figure 7](image-url)  
**Figure 7.** Kinematic model of the robot’s leg.
Using the analysis approach of [22], the coordinates of point \( \mathbf{O} \) will be used to define the displacement of the robot after a gait cycle. Its location can be obtained by finding the intersection of the actuator neutral axis \( z_n \) with the actuator fixed end.

Now, consider a four-bar linkage with link lengths calculated from the robot’s geometry such that

\[
a = |\mathbf{A} - \mathbf{O}|, \quad b = |\mathbf{B} - \mathbf{C}|, \quad g = |\mathbf{C} - \mathbf{O}|, \quad h = |\mathbf{B} - \mathbf{A}|.
\]

(12)

Using the analysis approach of [22], the coordinates of point \( F \mathbf{L} = (X, Y)^T \), expressed in base frame \( F \) is given by

\[
\begin{bmatrix}
X(\theta) \\
Y(\theta) \\
1
\end{bmatrix}
= \begin{bmatrix}
\cos(\theta + \phi) - \sin(\theta + \phi) \cos\theta \\
\sin(\theta + \phi) \cos(\theta + \phi) \sin\theta \\
0 \\
0 \\
1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix},
\]

(13)

where the coupler angle is

\[
\phi(\theta) = \arctan\left( \frac{b \sin \psi - a \sin \theta}{g + b \cos \psi - a \cos \theta} \right) - \theta,
\]

(14)

and

\[
\psi(\theta) = \arctan\left( \frac{B}{A} \right) \pm \arccos\left( \frac{C}{\sqrt{A^2 + B^2}} \right),
\]

\[
A(\theta) = 2ab \cos \theta - 2gb,
\]

\[
B(\theta) = 2ab \sin \theta,
\]

\[
C(\theta) = g^2 + b^2 + a^2 - h^2 - 2ag \cos \theta,
\]

(15)

\( \theta \) is the angle of the input link and \( (x, y)^T \) is the coordinates of a point \( \mathbf{L} \) on the robot’s leg frame \( \mathbf{M} \).

Next, define a contact frame \( W \) at \( \mathbf{L} \) and a body frame \( V \) at \( \mathbf{O} \), with both x-axis parallel each other. Let \( \beta \) be the angle between frame \( V \) and \( F \). Now, the coordinate of \( \mathbf{L} \) in frame \( V \) is then given by

\[
V_{L}(\theta) = \begin{bmatrix}
L_x(\theta) \\
L_y(\theta)
\end{bmatrix} = \begin{bmatrix}
\cos \beta - \sin \beta \\
\sin \beta \cos \beta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}.
\]

(16)

If the relationship is inverted with contact frame \( W \) being the fixed reference frame, the displacement of \( \mathbf{O} \) can be obtained as

\[
W_{O}(\theta) = -V_{L}(\theta) = \begin{bmatrix}
-L_x(\theta) \\
-L_y(\theta)
\end{bmatrix}.
\]

(17)

This defines the robot trajectory due to the leg contact at \( \mathbf{L} \) as it moves from Fig. 6(a) to (c). Hence, the displacement of \( \mathbf{O}_d \) and \( \mathbf{O}_c \), which corresponds to the robot displacement at \( t = 0 \) and \( t = T/2 \) can be obtained using

\[
\mathbf{O}_d = wO(\theta_{\min}),
\]

(18)

\[
\mathbf{O}_c = wO(\theta_{\max}.
\]

(19)

As shown in Fig. 6(d), the robot will return back to double leg stance due to gravity, by a clockwise rotation of \( \psi' \) pivoted about \( \mathbf{L} \). To determine the overall robot displacement, consider a parallel frame \( V' \) located on the rear leg piezoelectric actuator. Its origin is at \( \mathbf{O}' \) and its x-axis parallel to that of \( V \) as shown in Fig. 8. Define another contact frame \( W' \) with its origin at \( \mathbf{L'} \) and its x-axis parallel to that of frame \( V' \). Since the rear leg design is symmetrical to the front leg, the coordinate of \( \mathbf{L}' \) as seen in frame \( V' \) is an exact mirror image of its front leg with initial angle \( \theta_0 \). Thus, from Eq. (16), this yields

\[
V'_{L'} = \begin{bmatrix}
-L_x(\theta_0) \\
L_y(\theta_0)
\end{bmatrix}.
\]

(20)

To relate this back to the contact frame \( W \), consider the vector sum

\[
w_{L'}(\theta) \approx wO(\theta) + V'_{L'} = \begin{bmatrix}
d \\
0
\end{bmatrix} + \begin{bmatrix}
-L_x(\theta_0) \\
L_y(\theta_0)
\end{bmatrix}.
\]

(21)

Since the rotated angle \( \psi' \) in Fig 6(c) is very small, we can approximate it from Eq. (21) using trigonometry as

\[
\tan\psi' \approx \psi' = \frac{L_x(\theta_0) - L_x(\theta_{\max})}{d - L_x(\theta_0) - L_x(\theta_{\max})}.
\]

(22)

Now, to determine the pivot displacement \( \mathbf{O}_d \) in Fig. 6(d), we rotate the pivot displacement \( \mathbf{O}_c \) at gait instance Fig. 6(c) about the robot leg contact point \( \mathbf{L} \) by an angle \( \psi' \). This yields

\[
\mathbf{O}_d = \begin{bmatrix}
\cos \psi' \\
\sin \psi'
\end{bmatrix} \mathbf{O}_c.
\]

(23)
The overall robot displacement $S_{nad}$ for the front leg stance cycle can then be obtained as the difference between the displacement of $O$ from its contact frame $W$ at gait instance Fig. 6(a) and Fig. 6(d) respectively. This gives

$$S_{nad} = O_d - O_a.$$  \hspace{1cm} (24)

### 4.2 Back Leg Stance Gait Analysis

To determine the robot displacement during the back leg stance half, note that the contact point of the robot with the ground now shifts to the rear leg at $L'$ as shown in Fig. 6(e) to (f). Hence, for this subsequent gait cycle, we determine the displacement of pivot $O$ with frame $W'$ acting as the fixed reference frame. To express the displacement of pivot $O_d$ in frame $W'$, we use Eq. (21) and consider the vector sum

$$w'O_d = w'L + O_d = -\begin{bmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{bmatrix} \{L'_{\theta_{\max}} + O_d \}.$$  \hspace{1cm} (25)

Following similar analysis leading to Eq. (22) as described in the earlier section, we can approximate $\gamma$ as

$$\gamma = \frac{L_o(\theta_0) - L_g(\theta_{\min})}{d - L_x(\theta_0) - L_x(\theta_{\min})}.$$  \hspace{1cm} (26)

Since the rear leg is non-actuated and act to support the robot at $L'$ as illustrated in Fig. 6(d) to (f), there is only articulated motion at the front leg and no movement will occur on the robot’s body. Hence

$$O_f = O_e = w'O_d.$$  \hspace{1cm} (27)

Similarly, the robot will return back to double leg stance due to gravity, this time by an anti-clockwise rotation of $\gamma$ pivoted at $L'$ as shown in Fig. 6(g). Then, the displacement of pivot $O$ in frame $W'$ at this instance is

$$O_g = \begin{bmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{bmatrix} O_f.$$  \hspace{1cm} (28)

Now, the overall robot displacement $S_{dg}$ during the rear leg stance cycle can be obtained as the difference between the displacement of pivot $O$ from its contact frame $W'$ at gait instance Fig. 6(d) and Fig. 6(g) respectively. This gives

$$S_{dg} = O_b - w'O_d.$$  \hspace{1cm} (29)

### 4.3 Combined Gait Analysis

The total displacement of the robot after one full gait cycle as denoted in Fig. 6 can be calculated as

$$S_{gait} = \left\{ \begin{array}{c} S_x \\ S_y \end{array} \right\} = S_{nad} + S_{dg}.$$  \hspace{1cm} (30)

Upon substituting Eq. (18), (23), (25), and (28) into the above equation and simplifying the result, the average fore-aft speed of the robot given the applied frequency $(f)$ of the piezoelectric actuator can be calculated as

$$V_{bot} = f \cdot S_x = f \cdot \left| L_x(\theta_{\min}) - L_x(\theta_{\max}) \right| = f \cdot \left| L_x(\theta_{\min}) - L_x(\theta_{\max}) \right|.$$  \hspace{1cm} (31)

$\gamma$ and $\gamma'$ are given in Eq. (26) and Eq. (22) respectively, and $L_x(\theta)$ and $L_y(\theta)$ are given in Eq. (16). In fact, when $\gamma$ and $\gamma'$ are very small, the average linear speed of the robot based on this gait cycle can be easily approximated even further by

$$V_{bot} \approx f \cdot \left| L_x(\theta_{\min}) - L_x(\theta_{\max}) \right|.$$  \hspace{1cm} (32)

### 5 ROBOT DESIGN AND EXPERIMENTAL RESULTS

The piezoceramic material used for the robot is the soft-doped PZT (lead zirconate titanate) NCE55 from Noliac Inc. [23] and the elastic material is aluminium. Table 1 summarizes the properties of the materials used for the piezoelectric unimorph actuator.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>NCE55</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness, $t_p/l_m$ (mm)</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>Length, $l$ (mm)</td>
<td>25</td>
<td>31</td>
</tr>
<tr>
<td>Width, $b$ (mm)</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Density, $\rho_p/\rho_a$ (kgm$^{-3}$)</td>
<td>8000</td>
<td>2700</td>
</tr>
<tr>
<td>Elastic Compliance, $S_{11}^E$ (Pa$^{-1}$)</td>
<td>$17 \times 10^{-12}$</td>
<td>-</td>
</tr>
<tr>
<td>Piezoelectric Constant, $d_{33}$ (mV$^{-1}$)</td>
<td>$-260 \times 10^{-12}$</td>
<td>-</td>
</tr>
<tr>
<td>Young’s Modulus, $c_{in}$ (Pa)</td>
<td>-</td>
<td>$69 \times 10^9$</td>
</tr>
</tbody>
</table>
chosen solution is $C(8)$. They are as listed in Table 2. This yields 4 solutions and the solutions are obtained using the forward kinematic equations in Eq. (8). Due to manufacturability reasons, we choose an amplification factor of $\alpha = 3.5$ to obtain the range of $\theta_2$ using Eq. (9). Now, the task positions for the desired leg motion are obtained using the forward kinematic equations in Eq. (8). They are as listed in Table 2. This yields 4 solutions and the chosen solution is $C = (2.5, 2.83)$ and $B^1 = (-40.28, -4.50)$. The synthesized geometric parameters of the chosen four-bar linkage is as summarized in Table 3.

**TABLE 3.** The chosen and synthesized geometric parameters of the robot.

<table>
<thead>
<tr>
<th>a = 36.64 mm</th>
<th>b = 43.4 mm</th>
<th>g = 3.78 mm</th>
<th>h = 4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 2.33 mm</td>
<td>y = 18.33 mm</td>
<td>$\beta = 48.56^\circ$</td>
<td>$\theta_0 = 136.4^\circ$</td>
</tr>
</tbody>
</table>

To verify the validity of the robot’s kinematic model, the robot’s body and legs are fabricated fully using 3D printer with ABS material. The length of the aluminium is 3 mm longer than the length of the piezoelectric at each side to enable it to be clamped onto the printed robot using standard screws. Fig. 9 shows the fabricated robot prototype. The two legs at each end of the robot are connected together at the bottom so that they produce similar gait to ensure straight motion. The final dimension of the robot prototype is 100 x 17.4 x 27.2 mm and it weighs 12.17 g.

**5.2 Kinematic Model Verification**

Experiments were conducted to find out the experimental speed of the robot and compare it with the speed calculated using our kinematic model. The robot was let to run in one direction on a flat wooden surface for a distance of 30 cm, and the time taken to complete the task was noted down. The robot’s speed was calculated by dividing the distance travelled by the time taken. An image sequence of the piezoelectric robot in motion during one of the trials is shown in Fig. 10.

The first resonant frequency of the actuator is obtained theoretically as 917.8 Hz using Eq. (5), which is a useful guide to experimentally attain the first resonant frequency of the actuator. Experiments were carried out on the robot prototype by varying the frequency of a 100 V input from 600 Hz to 900 Hz at an interval of 50 Hz. Five trials were performed for each frequency and the results are plotted in Fig. 11. The data points show the mean results while the shaded areas show that maximum and minimum values of the trials. The first resonant frequency of the actuator was found experimentally to be equal to 800 Hz, and this would be the constant applied frequency used to validate our model.

The experiment and analytical speed calculation were then performed at four different applied voltages: 40 V, 60 V, 80 V, and 100 V. Similarly, the experiments were repeated five times for each applied voltage and the results are plotted in Fig. 12. The robot is able to achieve an average maximum speed of 3.97 cm/s while consuming 0.8 W in doing so. To analytically determine the dynamic displacement, an experimental dynamic factor $Q$ needs to be multiplied to the displacement $w(x)$ produced in static operation. This yields a dynamic displacement of

$$w_d(x) = w(x) \ast Q.$$  \hspace{1cm} (33)

The scalar $Q$ can be experimentally determined [24, 25] and its value will be different for each applied frequency $f$. At the first resonant frequency, $Q$ was experimentally found to be 0.32. From the plot, it can be seen that the behaviour of the robot prototype corresponds closely to the result as predicted from our kinematic model. The slight difference could be due to the frictional loss in the rotational joints or leg slippage during locomotion, which was not taken into account.

**FIGURE 9.** Prototype of the fabricated miniature piezoelectric robot.
FIGURE 10. Image sequence of the piezoelectric robot in one of the trials.

FIGURE 11. Plot of the robot’s speed vs applied frequency on a flat surface.

FIGURE 12. Plot of the robot’s average speed vs applied voltage on a flat surface.

FIGURE 13. Concept of the future multi degree of freedom robot.

6 CONCLUSIONS
The design and theoretical analysis of a new type of piezoelectric legged robot with a four-bar architecture is presented in this paper. Its gait cycle is described and kinematic modelling is performed to estimate its speed based on an input voltage. A robot prototype is fabricated to test the validity of the derived kinematic model and it is found that the behaviour of the physical prototype corresponds with the model. The derived model will be useful for controlling and planning the motion of the robot based on a control input. The end goal of this work is to have a controllable multi-degree-of-freedom miniature robot using piezoelectric actuators. As such, we have also in mind the concept of a multi-degree-of-freedom quadrupedal robot as shown in Fig. 13, whereby each leg will drive the robot towards different direction, as the next iteration of this work.

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REFERENCES


