Steerable Miniature Legged Robot Driven by a Single Piezoelectric Bending Unimorph Actuator

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Abstract—In small mobile robots, decreasing the number of actuators is usually desirable to reduce the size and weight of the robot, but it is usually at the expense of the robot’s degree of freedom (DOF). This work presents the development and preliminary experimental testing of a novel Legged Piezoelectric Miniature Robot (LPMR) driven only by a single piezoelectric unimorph actuator and yet fully capable of being maneuvered to move forward, turn right, or turn left. The underactuated motion is achieved by exploiting the bending vibration modes disparity of the piezoelectric actuator at different driving frequencies and designing specific positions of the robot’s legs to generate a differential-drive-like mechanism. The speed of the robot can be controlled through regulating the magnitude of the applied voltage. The proposed underactuated system is experimentally verified and a preliminary characterization of the LPMR in terms of its forward and turning speed versus applied voltage and payload is investigated and reported.

I. INTRODUCTION

Miniature mobile robots exhibit various benefits over their larger counterparts. Due to their small size, they are able to fit into narrow spaces that are inaccessible to humans or large robots to aid in search and rescue efforts in disaster sites where hazardous environment such as debris, extreme temperatures, and chemical toxicity may be present, or simply in inspection or reconnaissance operations in remote places. They are also easily transported and deployed into the constrained locations due to their light weight.

Motivated by these advantages, a variety of miniature robots have been developed by several groups of researchers. As the key elements of a miniature robot are its size and weight, most of the efforts have been invested in developing and investigating minimally or underactuated robots to reduce the number of robot’s components, thus achieving miniaturization. Mobility is another important aspect and the robot also needs to be maneuverable despite it being minimally or under-actuated. Various actuation methods and locomotion principles have been proposed and applied.


For the locomotion, DASH, X2-VelociRoACH, as well as the robots developed by Yumaryanto et al. and Rios et al. employ alternating tripod gait. HAMR-VP and Goldfarb et al. make use of trotting gait. Ho et al. implements bounding gait. Kilobot applies the stick-slip locomotion principle.

Most of the steering techniques employed are based on the differential drive mechanism which can be achieved through various methods, for example controlling the right and left sides of the robot independently [5], [7], [11], [12], [13], controlling the drive and lift DOF of the legs independently [14], [3], altering the leg stiffness [15], skewing the robot’s body frame [9], adding electroadhesive foot pads [16], and shifting the center of mass of the robot [17]. Other steering techniques include producing an angular momentum on the robot’s body through the dynamic motion of a weighted tail [18] or attaching a sail at the tip of the tail [19].

As can be seen from the examples above, all of the robots require at least two actuators to achieve straight and turning motion. The first robot actuated by a single actuator which can be driven straight and steered is ISTAR [20], [21]. The robot exploits the compliant disparity between the alternate stance tripods to generate body rotation by continuously accelerating and decelerating the legs which is controlled by a single motor. In this paper, we propose an alternative which differs from above, and describe the design and characterization of an underactuated miniature robot architecture, shown in Fig. 1, that is capable of planar motion. The robot is propelled by a piezoelectric unimorph actuator at three different modal frequencies to generate the
forward and steering motion of the robot.

This work represents the first step in applying and verifying the efficacy of the proposed locomotion principle to further shrink miniature robots while preserving its functionality. The operating principle of the robot is described in section II. In section III, we discuss the physical robot design and fabrication. The experimental results are presented in section IV. Section V discusses the current advantages and limitation of the robot as well as possible future works.

II. LOCOMOTION PRINCIPLE

The fundamental locomotion principle of our robot is based on the working principle of the robot in [22] which exploits the bending wave of the piezoelectric actuator to achieve bidirectional bounding gait locomotion. Essentially, legs located on the left of the antinode propel the robot towards the right and vice versa. Fig. 2 shows the virtual representation of the locomotion of one gait cycle when the legs are positioned at the different sides of the antinodes.

The steering mechanism of the robot is inspired by differential drive system. In differential drive, the velocities of the right and left side of the mobile robot are different, resulting in a turning moment of the robot’s body. In our robot, we can achieve this by placing one robot’s leg slightly to the left of the antinode and the other leg slightly to the right of the antinode at two opposite sides of its midsagittal plane, as depicted in Fig. 3. Hence, with quadrupedal robot, moving forward, steering left, and steering right can be achieved by carefully choosing the locations of the robot’s legs that generate the three different motions at three different bending modes, which will be discussed further in the next section.

III. ROBOT DESIGN & FABRICATION

A. Leg Location Design

To generate a planar motion, three bending modes of the piezoelectric actuator are utilized: the first resonance frequency \( f_1 \) for straight motion, the second \( f_2 \) for left steering, and the third \( f_3 \) for right steering. Hence, for a quadrupedal robot, the leg design requirements are: one pair of the legs is positioned to produce the left turning moment at \( f_2 \) but unidirectional motion at \( f_1 \) and \( f_3 \) to help the robot to move forward simultaneously, while the other pair is positioned to generate the right turning moment at \( f_3 \) but unidirectional motion at \( f_1 \) and \( f_2 \).

Fig. 4 shows the bending modes of the piezoelectric actuator at \( f_1 \), \( f_2 \), and \( f_3 \), color-coded in red, blue, and black respectively. The mode shapes are analytically obtained from Eq. 19 in [23]. The possible positions of the legs near the antinodes are shaded in purple for \( f_2 \) and brown for \( f_3 \). The direction of motion produced by the pair of legs positioned at the various antinodes is summarized and shown by the arrows at the bottom of the graph color-coded based on the applied frequency. Antinode \( f_{3b} \) is out of the question because the legs positioned there do not produce unidirectional motion at \( f_1 \). The rest of the antinodes \( (f_{3a}, f_{2a}, f_{2b}, \text{and } f_{3c}) \) all meet the design requirements discussed in the previous paragraph. Hence, a pair of the legs can be positioned at either \( f_{2a} \) or \( f_{2b} \), while the other pair at either \( f_{3a} \) or \( f_{3c} \).

Fig. 4: Bending modes of the piezoelectric actuator at \( f_1 \), \( f_2 \), and \( f_3 \), color-coded accordingly. The possible positions of the pair of legs near the antinodes are shaded. The arrows at the bottom denote the direction of motion produced by the legs at each particular frequency.
It may be intuitive to position the legs at a combination of either \( \{ f3a - f2a \} \) or \( \{ f2b - f3c \} \) since these combinations produce the same direction of unidirectional motion at \( f_1 \). However, positioning the two pairs of legs close to each other and concentrated only at one half of the robot’s body length is detrimental to the robot’s balance. Hence, combination \( \{ f3a - f2b \} \) or \( \{ f2a - f3c \} \) should be chosen instead. At these combinations, we expect the robot to either be stationary at \( f_1 \) if the opposite unidirectional forces cancel each other perfectly or still progress towards one direction if one of the unidirectional forces is stronger than the other. The experimental results show that the robot can still progress forward at \( f_1 \). The leg combination used in the experiment is \( \{ f3a - f2b \} \). Fig. 5 summarizes the effects of this leg configuration at the three different resonance frequencies and the expected resultant robot motion.

![Fig. 5: Effects of the legs and expected resultant motion of the robot at three different resonance frequencies.](image)

### B. Actuator Design

The design of the actuator requires the sizing and the choice of the elastic material used, such as aluminium, brass, steel, or acrylic. To choose the appropriate material, we employ the analytical equation elaborated in [24] to calculate the expected tip displacement of the unimorph actuator, and equation in [25] to determine the expected blocking force which is the force required to be applied at the tip of the actuator to resist the deflection. The maximum displacement of the unimorph actuator is calculated as

\[
\delta = \frac{M_{eq} L^2}{2 (EI)_{eq}},
\]

while the blocking force is calculated as

\[
F_b = \frac{3 M_{eq}}{2 L},
\]

where

\[
M_{eq} = -qd_{31} S_{11},
\]

\[
(EI)_{eq} = (\frac{I_p}{S_{11}} + I_m c_m),
\]

\[
q = \frac{1}{2} b[(t_p + t_m - z_n)^2 - (t_m - z_n)^2],
\]

\[
I_p = \frac{1}{3} b[(t_p + t_m - z_n)^3 - (t_m - z_n)^3],
\]

\[
I_m = \frac{1}{3} b[(t_m - z_n)^3 + z_n^3],
\]

\[
z_n = \frac{1}{2} \frac{c_m t_m^2 + c_p t_p t_m}{c_m t_m + c_p t_p},
\]

\( L \) is the length of the actuator, \( b \) is the width of the actuator, \( t_m \) is the thickness of the elastic material, \( t_p \) is the thickness of the piezoelectric layer, \( d_{31} \) is the piezoelectric constant, \( S_{11}^E \) is the elastic compliance of the piezoelectric material, \( E_3 \) is the applied electric field, and \( c_m \) is the Young’s modulus of the respective elastic material.

Fig. 6 shows the actuator deflection and the blocking force for the various materials when their thickness are varied. Although the metallic materials have large deflection at 0.2 mm and thus are expected to deliver the highest speed, their blocking forces at this thickness are low. We want to balance between large deflection and high blocking force so that the deflection of the actuator is less affected by external force. Hence thickness between 0.5 mm and 1.0 mm are chosen. For the material, aluminium is chosen as it has the largest deflection at these thickness and the lowest density amongst the three metallic materials. For experiment, two robots were fabricated: one using 1.0 mm aluminium while the other using 0.5 mm aluminium as the elastic material. This is also to verify that the thinner aluminium would produce higher speed as the deflection of the actuator is expected to be larger based on the analytical calculation. Table I summarizes the properties of the materials used for the piezoelectric unimorph actuator, which consist of piezoelectric material NCE41 from Noliac Inc. and Aluminium 6061.

![Fig. 6: Actuator deflection and blocking force versus elastic material thickness.](image)

### C. Fabrication

The piezoelectric patch and the aluminium beam are joined together using a strong bonding epoxy adhesive (two-part Araldite epoxy). Four cylindrical-shaped steel legs are bonded onto the aluminium beam using the same epoxy at the desired locations based on the leg position design in section III-A. Fig. 7 summarizes the positions of the legs along the body length of the robot measured from one end.
### TABLE I: Properties of the actuator materials.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>NCE41</th>
<th>Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness, $t_p/t_m$ (mm)</td>
<td>0.5</td>
<td>1.0 and 0.5</td>
</tr>
<tr>
<td>Length, $L$ (mm)</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Width, $b$ (mm)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Density, $\rho_p/\rho_m$ (kgm$^{-3}$)</td>
<td>7900</td>
<td>2700</td>
</tr>
<tr>
<td>Elastic Compliance, $S_{11}$ (Pam$^{-1}$)</td>
<td>$13 \times 10^{-12}$</td>
<td>-</td>
</tr>
<tr>
<td>Piezoelectric Constant, $d_{31}$ (mV$^{-1}$)</td>
<td>$-130 \times 10^{-12}$</td>
<td>-</td>
</tr>
<tr>
<td>Young’s Modulus, $c_m$ (Pa)</td>
<td>-</td>
<td>$69 \times 10^9$</td>
</tr>
</tbody>
</table>

Fig. 7: Bottom view of the robot and the position of the robot’s legs along the body length.

of the actuator. The positions of the legs are as close to the actuator’s edge as possible on the desired side. Finally, thin wires are soldered onto the piezoelectric patch. The prototype’s dimension is $50 \times 10 \times 9$ mm (excluding cables) and the mass is $3$ g for the robot using $0.5$ mm aluminium, while the mass of the robot with $1$ mm aluminium is $4$ g and its height is $9.5$ mm due to the thicker material.

### IV. EXPERIMENTAL RESULTS

Experiments were conducted to verify the behaviour of the robot and to characterize its performance. The natural frequencies of the robots are calculated analytically using Eq. 18 elaborated in [23] which can be expressed as

$$f_n = \frac{B_n l}{2\pi L^2} \frac{EI_{eq}(\rho_p t_p + \rho_m t_m)}{b(\rho_p t_p + \rho_m t_m)}, \quad (3)$$

where $n$ denotes the $n^{th}$ resonance frequency, $\rho_p$ and $\rho_m$ are the density of the piezoelectric and elastic material respectively, and $B_n l$ is the solution to the characteristic equation for free-free piezoelectric unimorph actuator and is given as 4.73, 7.85, and 10.99 for the first three natural frequencies respectively. In calculating the theoretical resonance frequencies of the system, the inertia contribution of additional mass, such as legs or payload, are not accounted for. Table II summarizes the analytical resonance frequencies of the robots which are used in the experiment to control the motion of the robot.

<table>
<thead>
<tr>
<th>Resonance Frequency</th>
<th>1.0 mm aluminium</th>
<th>0.5 mm aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ (kHz)</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$f_2$ (kHz)</td>
<td>6.9</td>
<td>4.2</td>
</tr>
<tr>
<td>$f_3$ (kHz)</td>
<td>13.5</td>
<td>8.2</td>
</tr>
</tbody>
</table>

**TABLE II: Resonance frequencies of the robots.**

For the experiments, the robot run on a glass surface for a certain amount of time and the distance covered was noted down to compute the average speed of the robot. For the left and right steering, the forward and sideward displacement were noted down separately to obtain the forward and sideward speed of the robot independently. As the left and right steering motion of the robot were observed as close to being linear (see accompanying video attachment), the resultant speed of the robot and the average steering angle can be calculated from the forward and sideward displacement.

Two experiments were performed. First, the robot’s speed was measured as the applied voltage was varied from $20$ V to $140$ V in a $20$ V increment while the payload was kept constant at $10$ g. Second, the speed was measured as payload was added to the robot from $10$ g to $100$ g in a $10$ g increment while the applied voltage was kept constant at $140$ V. The added payload was in the form of cylindrical masses placed on top of the robot’s body. For each applied voltage and payload, the experiment was repeated five times, and all the experimental results are plotted in Fig. 8 and Fig. 9.

Fig. 10 further illustrates the locomotion direction of the robot for $f_2$ and $f_3$, which is depicted as a straight interpolation between the start and end position, while Fig. 11 summarizes the average steering angle of the robots which is defined as the deviation angle of the robot’s locomotion direction from its initial forward orientation.

Fig. 8: Linear speed of the robot versus applied voltage.

Fig. 9: Linear speed of the robot versus payload.

### V. DISCUSSION

As seen in Fig. 8 and 9, the robot’s speed behaves as anticipated where it increases as the applied voltage increases.

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**Parameter (unit) NCE41 Aluminium
Thickness, $t_p/t_m$ (mm) 0.5 1.0 and 0.5
Length, $L$ (mm) 50 50
Width, $b$ (mm) 10 10
Density, $\rho_p/\rho_m$ (kgm$^{-3}$) 7900 2700
Elastic Compliance, $S_{11}$ (Pam$^{-1}$) $13 \times 10^{-12}$ -
Piezoelectric Constant, $d_{31}$ (mV$^{-1}$) $-130 \times 10^{-12}$ -
Young’s Modulus, $c_m$ (Pa) - $69 \times 10^9$**
and decreases as the payload increases. The speed of the robot with 0.5 mm aluminium is always higher than the robot with 1.0 mm aluminium at each resonance frequency, which supports the analytical calculation of the actuator design in section III-B. It can also be observed that the relationship between the robot’s speed and the voltage approximately follows a linear trend with different slopes.

In Fig. 10 (top) and 11 (top), it can be observed that the steering angle is irregular as the voltage is varied. This might be the effect of fluctuating slip between the robot’s legs and the ground. As the robot’s speed increases, slip is more likely to happen due to higher horizontal force component at the tip of the robot’s leg. When slip occurs, especially at the pair of legs which is responsible for maneuvering the robot, the steering direction will be affected. The addition of payload helps to increase friction between the robot’s legs and the ground, and hence the steering angle of the robots is relatively more consistent as payload is added onto the robot, as observed in Fig. 10 (bottom) and 11 (bottom).

It is also of interest to know the power consumption of the robots. The power consumption of the robot can be analytically calculated using

\[ P = \frac{1}{2} f CV^2 \] (4)

where \( f \) is the applied frequency, \( C \) is the capacitance of the piezoelectric patch (10944\(p\)F for NCE41), and \( V \) is the applied voltage. The highest power consumption of the robots, which occurs at 140 V and when the robot is used for right steering (\( f3 \)), is 1.45 W and 0.88 W for the robot using 1.0 mm aluminium and 0.5 mm aluminium respectively.

VI. CONCLUSION

With the data available, we are able to perform comparison against several existing miniature robots in terms of the parameters that are usually reported in the literature, such as robot’s dimension, mass, top speed, type of actuators used, number of actuators, and steering capability. The existing miniature robots are compared against our robot with 0.5 mm aluminium as it gave the better performance. For the mass of the robot, we added the mass of the battery and circuit board that we are currently developing for a power-autonomous version of the robot in the future, which is expected to be roughly 14.5 g. Table III summarizes the parameters of some comparable existing legged miniature robots.

From the table, the robot developed in this work is the tiniest and lightest maneuverable single-actuator miniature robot to the best of the authors’ knowledge. Its speed in terms of bodylength/second is comparable to the other steerable single-actuator robot, 1STAR [20], which is about 2.4 times bigger in size and 4.5 times heavier. In the future, further optimization of the robot will be investigated to bring down the robot’s size and weight as well as increasing the robot’s speed and motion precision. This includes optimization of the piezoelectric material, in-depth investigation of the actuator’s bending modes and leg’s slip, and a more delicate and accurate fabrication process. Development of a tetherless version of the robot will also be part of our future work.
TABLE III: Comparison to Existing Legged Miniature Robots.

<table>
<thead>
<tr>
<th>Dimension (mm)</th>
<th>Mass (g)</th>
<th>Top Speed (cm/s)</th>
<th>Actuator Type</th>
<th># of Actuators</th>
<th>Steering Capability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l x w x h)</td>
<td></td>
<td>-(bodylength/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Our Robot</strong></td>
<td>50 x 10 x 9</td>
<td>17.5 ¹</td>
<td>14 (2.8)</td>
<td>Piezoelectric</td>
<td>1</td>
</tr>
<tr>
<td>Goldfarb et al. [5]</td>
<td>90 x 65 x 50</td>
<td>104</td>
<td>30 (3.3)</td>
<td>Piezoelectric</td>
<td>2</td>
</tr>
<tr>
<td>HAMR-VP [3]</td>
<td>44 ²</td>
<td>1.27 ³</td>
<td>44 (10.1)</td>
<td>Piezoelectric</td>
<td>6</td>
</tr>
<tr>
<td>Qu et al. [4]</td>
<td>36.4 ²</td>
<td>4.4 ⁴</td>
<td>0.4 (0.1)</td>
<td>Piezoelectric</td>
<td>6</td>
</tr>
<tr>
<td>Rios et al. [13]</td>
<td>55 ²</td>
<td>16 ³</td>
<td>52 (9.5)</td>
<td>Piezoelectric</td>
<td>12</td>
</tr>
<tr>
<td>Yumaryanto et al. [6]</td>
<td>135 x 55 x 65</td>
<td>55 ²</td>
<td>18 (1.3)</td>
<td>LIPCA</td>
<td>2</td>
</tr>
<tr>
<td>Ho et al. [7]</td>
<td>120 x 80 x 70</td>
<td>135</td>
<td>22 (1.8)</td>
<td>LIPCA &amp; servo</td>
<td>4</td>
</tr>
<tr>
<td>Kilobot [8]</td>
<td>33 x 33 ⁵</td>
<td>Not available</td>
<td>1 (0.3)</td>
<td>Vibration motor</td>
<td>2</td>
</tr>
<tr>
<td>DASH [9]</td>
<td>100 x 50 x 100</td>
<td>16.2</td>
<td>150 (15)</td>
<td>Motor</td>
<td>2</td>
</tr>
<tr>
<td>VelociRoACH [10]</td>
<td>104 x 64 x 49</td>
<td>54.6</td>
<td>490 (47)</td>
<td>Motor</td>
<td>1</td>
</tr>
<tr>
<td>ISTAR [20]</td>
<td>120 ²</td>
<td>80</td>
<td>35 (2.9)</td>
<td>Motor</td>
<td>1</td>
</tr>
</tbody>
</table>

¹Expected mass of the power-autonomous version of the robot that we are currently developing. The mass of the robot without onboard electronics and battery is 3 g.
²Only body length is available.
³Excluding onboard electronics and battery.
⁴Only body diameter is available.
⁵Only body diameter is available.

References


